Department:Math **Section: Math / Phy.- Phy./Chem.- Phy.**

Year : fourth - third **Term: Second**

Course Title: Math. Programming and Numerical Methods

Course Code: Math 350 **Total Points: 70**

Dr. Faisal Ezz Eldeen **Date: 31 / 5 / 2016**

Question 1

1- Find the **first two iterations** of the **Jacobi** method for the following linear system using, $x^{(0)} = y^{(0)} = z^{(0)} = 0$:

$$5x - y + 3z = -2,$$
$$x + 5y - 2z = 10,$$
$$2x - 4y + 10z = 6$$

2- Write a computer program to find $\int_1^2 x \ln x \, dx$, $n = 4$ using Trapezoidal rule.

Question 2

1- Giving the following table:

x	1.0	1.3	1.6	1.9
$f(x)$	0.76519770	0.62008600	0.45540220	0.28181860

(i) Use a **Lagrange** interpolating polynomial of degree **two** to approximate $f(1.5)$.

(ii) Approximate $f'(1.9)$ and $f'(1.6)$.

2- Write a computer program to obtain the solution of $x - \cos x = 0$ lying in the interval $\left[0, \frac{\pi}{2}\right]$, using Bisection method.

Question 3

1- Use Euler's method to approximate the solutions of the initial- value problem

$y' = y - x^2 + 1$ for $0 \le x \le 2$ with $y(0) = 0.5$, $n = 10$.

2- Use Newton's method with $x_0 = 2$, $y_0 = -2$ to find the solution of the nonlinear system (**one iteration only**):

$$x^2 + y = 3,$$
$$x + y^2 = 5.$$

ملاحظة هامة: الباقي خلف الصفحة

Question 4

1-Write a computer program to solve the initial value problem $y' = y - x^2 + 1$ for $0 \leq x \leq 2$ with $y(x_0) = 0.5, n = 10$ using Runge- Kutta method.

2- Use the Simpson's rule to approximate the following integral

$$\int_0^1 \frac{1}{1+x^2}\, dx, \quad n = 4.$$

Good luck

Course Code: M352
Course Title: Fluid Dynamics (I)
Year: third
Section: Physics/Math
Term: second
Time Allowed: 2 hours
Total Points: 70.
Date: 1/6/2016
Dr.: Shreen Elsapa

Damanhour University
Faculty of Science
Department of Mathematics

Answer the Following Questions

1) The velocity $\vec{q}$ of an incompressible two-dimensional flow is given by $\vec{q} = -\nabla\phi$ where $\phi = (x-t)(y-t)$.

(a) Show that the streamlines at time t are the curves $(x-t)^2 - (y-t)^2 = const.$

(b) The paths of the particles have the equation

$$\ln(x-y) = \tfrac{1}{2}(x+y) - \frac{A}{x-y} + B,$$ where A, B are constants.

(c) Calculate the components of the acceleration at any position (x, y) for all t.

2) (a) Discuss physically and prove that the equation of continuity for steady incompressible fluid is $\nabla \cdot \vec{q} = 0$.

(b) Show that

$$q_x = -\frac{2xyz}{\left(x^2+y^2\right)^2}, \quad q_y = \frac{\left(x^2-y^2\right)z}{\left(x^2+y^2\right)^2}, \quad q_z = \frac{y}{x^2+y^2}$$

are the velocity components of a possible fluid motion. Is this motion irrotational?

3) A sphere of radius a is surrounded by a concentric spherical shell of radius b, and the space between is filled with liquid. If the sphere is moving with velocity V, show that

$$\phi = \frac{Va^3}{b^3-a^3}\left(r + \frac{b^3}{2r^2}\right)\cos\theta.$$

Calculate the kinetic energy of the liquid.

Please turn over

4) (a) Prove that Bernoulli's equation for irrotational motion, this equation becomes

$$\int_{p_o}^{p} \frac{dp}{\rho} + \tfrac{1}{2}q^2 + \Omega - \frac{\partial \phi}{\partial t} = C \quad \text{where } C \text{ is an absolute constant.}$$

(b) A sphere of radius a is moving with a non-uniform velocity U in an incompressible fluid which is otherwise at rest. The velocity of the fluid at the position $\vec{r}$ relative to the center of the sphere is given by $\vec{q} = -\nabla\phi$ where $\phi = \frac{a^3}{2r^3}\vec{U}\cdot\vec{r}$. If the resulting motion in the liquid is irrotational and p_o is the pressure at infinity, find the pressure at any point of the surface of the sphere. Hence, evaluate the resultant liquid thrust on the sphere.

With best wishes

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امتحانات قديمة لسنوات سابقة

جامعة الإسكندرية - كلية العلوم بدمنهور
الفرقة الثالثة (رياضيات) "ميكانيكا الكم"
دور يناير ٢٠١٠
الزمن: ٣ ساعات

أجب عن الأسئلة الآتية.

$(1-a)$ State the Plank's formula in the black body radiation and find the Wien's formula and Rayleigh-Jeans formula.

$(1-b)$ In the framework of quantal treatments, show the basic law of classical mechanics can be obtained as a special case.

$(2-a)$ Discuss the uncertainty principle and then prove the uncertainty relations and then prove uncertainty principle for the superposition state $\psi(x,t) = \frac{1}{\sqrt{2}} \{\psi_0(x,t) + \psi_1(x,t)\}$ of harmonic motion.

$(2-b)$ Obtain the eigenfunctions for a particle being subjected to an elastic restoring force of coordinate-dependent form (the linear harmonic oscillator).

(3) Find the eigenfunctions and the corresponding eigenvalues of a particle of mass m and energy $E > V_0$ existing in potential field $V(x) = V_0$ at $-a \leq x \leq a$, and $V(x) = 0$ at $x < -a,\ x > a$.

$(4-a)$ Obtain the eigenfunctions and the corresponding eigenvalues for the operator $\hat{L}^2$ using the spherical-polar coordinates.

$(4-b)$ A particle of mass m falls onto a horizontal plane and elastically bounces up. Quantize the particle's motion and determine the admissible heights and calculate the energy levels of this system.

(5) Write down Schrödinger's equation for two-body and transform it to relative and center of mass coordinates, and obtain the new form of Schrödinger's equation and then find its solution.

Alexandria university	Third year Exam (special math)
Faculty of science	M(305) Differential and integral equations
Damanhour	1st semester
Department of Math	1/2010 Time:3 hours

Answer five questions only

1) a- Define the Laplace transform of the function f(x).Hence show that

i) $L[\, e^{-ax} f(x)\,] = \overline{f}(p+a)$

ii) $L\,[\, H(x-a)Y(x-a)\,] = e^{-pa}\overline{Y}\ (p+a)$,

where $L[f(x)] = \overline{f}(p)$ and H(x) is the Heaviside step function.

b) Use the Laplace transforms, to solve the differential equation $y'' + 2y' + y = 3xe^{-x}$, given

$y(0) = 4,\ y'(0) = 2$

c) Evaluate: $L^{-1}\left[\frac{e^{-3p}}{(1+p)^3}\right]$, $L^{-1}\left[\frac{1}{p(p+1)^2}\right]$ (30 marks)

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2) Find two linearly independent solutions valid about x=0 for the differential equation

$$xy'' + (x+1)y' + 2y = 0.$$

(30 marks)

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3) a-Obtain the condition of integrability of the total differential equation

$$P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0$$

b- Solve the following:

i) $(2x^3y + 1)dx + x^4dy + x^2e^{-z}dz = 0$

ii) $yzdx - z^2dy - xydz = 0$

iii) $\frac{dx}{4y-3z} = \frac{dy}{4x-2z} = \frac{dz}{2y-3x}$

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4) a- Find the general solution of the differential equation $y'' - 6y' + 9y = \frac{1}{x^2}e^{3x}$

b- Show that sin x is a solution of the related homogeneous equation of

$y'' - (2tanx)y' + 3y = 2secx$, then find the general solution.

c- Put the differential equation $y'' - (2sinx)y' + (sin^2x + cosx - \frac{2}{x^2})y = e^{cosx}$

in the normal form , then find the general solution.

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5)a- Show that a linear (second order)differential equation with constant coefficients reduces under any initial conditions to a Voltera integral equation of the second kind .Hence form an integral equation corresponding to $y'' - y'sinx + ye^x = x$, $y(0) = 1,\ y'(0) = -1$

b-Solve $\varphi(x) = e^x + \int_0^x e^{x-t}\,\varphi(t)dt$

(30 Marks)

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6) Show that , if u(x) satisfies the integral equation $u(x) = f(x) + \int_0^x g(x-t)\,u(t)dt$.

Then $\bar{u}(p) = \frac{\bar{f}(p)}{1-\bar{g}(p)}$,

where $\bar{f}(p)$ and $\bar{g}(p)$ are the Laplace transforms of f(x) and g(x) respectively. Hence obtain the solution of the equation $u(x) = cosx + \int_0^x sin(x-t)\,u(t)dt$

Alexandria University Faculty of Science in Damanhour Department of Mathematics			
3rd year (Mathematics)			
Course title: Algebra			
Date:	Term: 1st 2010		Time:3 hours

Answer only 4 questions of the following:

1-If m is a positive integer and S= {1,2,3,.....m-1} , * is an operation on S defined by:

$a * b = a + b$ if $a+b < m$,

$= r$ if $a+b = m+r$.

Prove that (S , *) is a group .

Construct the multiplication table of (S , *) for m=4.

2-(a) Calculate $\alpha\beta, \beta\alpha, \alpha^{-1}$, where

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 6 & 4 & 5 & 1 \end{pmatrix}, \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 2 & 5 & 1 & 4 \end{pmatrix}$$

(b) Prove : The intersection of two subgroups H , W of a group (G , *) is a subgroup of G.

3-(a) Determine the even and odd permutations of the following:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 6 & 4 & 5 & 7 & 1 \end{pmatrix}, \delta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 2 & 5 & 1 & 4 & 8 & 7 \end{pmatrix}$$

(b) Find the elements of S_3 , determine A_3 , is A_3 a subgroup of S_3 ?

4-(a) Show that of even permutation A_n is a subgroup of S_n.

(b)Is the group S_n commutative?

5-(a) What do you mean by saying "σ is an isometry"? , give an example for an isometry and another example which is not an isometry.

(b)Let G be the set of real numbers of the form $a + b\sqrt{2}$,where

a, b are real is this form is a gp

Alexandria University Faculty of Science in Damanhour Department of Mathematics			
3rd year (Mathematics)			
Course title		Course code	
date	Term: Topology and real analysis		time

Answer five questions

4- (a) **Explain how did topology start**

(b)**Prove "The intersection of three topologies on a set X is a topology on X",**

Is the union of two topologies on a set X also a topology on X?

(c)**If X={a,b,c,d,e} , S={{a,b,c},{b,c,d},{c,d,e}},Find the topology on X generated by S and use the results to define and explain the concepts of subbase and base of a topology.**

5-(a) **IF X={a,b,c,d,e} and τ = {X, ϕ,{a},{c,d},{a,c,d},{b,c,d,e}} , A ={a,b,e} , find the limit points of A .**

(b) **-(a) Find the neighborhoods for all points o X ={a,b,c,d,e} , with respect to**

τ = {X, ϕ,{a},{c,d},{a,c,d},{b,c,d,e}}

(c)**Define , Closure , interior, boundary and exterior of a set , find these concepts for all subsets of A ={a,c,e} with respect to τ = {A, ϕ ,{a},{c},{a,c},{c,e}}**

6- **(a)- Explain an example for the application of topology in real life problems**

(b)**If (X, τ) is a topological space , Y X , prove that τ ={ Y U : U τ } is a topology on Y.**

(c)Prove " the intersection of two neighborhoods of a point *p* is a neighborhood of *p*.

Alexandria University First Term 2010
Faculty of Science (Damanhour) Third Year Exam
"Real Analysis" and "Topology" Time: 3 hours

Answer 5 questions only

(1) a) State and prove Hölder's inequality for integrals, hence deduce that

$$\int_a^b |y(t)|\,dt \le \sqrt{b-a}\left(\int_a^b |y(t)|^2\,dt\right)^{1/2},$$

$\forall\ y(t) \in L_2[a,b]$.

b) Show that the space $L_p[a,b]$ is a metric space.

(2) a) State and prove Minkowsk's inequality for sums.

b) Prove that the n-dimensional complex plane C^n is a metric space with the distance function $\rho(x,y) = \sqrt{\sum_{i=1}^{n} |\xi_i - \eta_i|^2}$, $\forall x = \{\xi_i\} \in C^n,\ y = \{\eta_i\} \in C^n$.

(3) a) Give a definition of complete metric spaces.

b) Give and discuss an example for an incomplete metric space and another which is complete.

c) Show that if $\{x_n\}$ and $\{y_n\}$ are two sequences of a metric space X such that

$x_n \to x$, $y_n \to y$ (as $n \to \infty$), then

$\rho(x_n, y_n) \longrightarrow \rho(x,y)$ as $n \to \infty$.

"With Best Wishes" Prof. Dr. Fathi Barakat

"تابع سير فضلك أسئلة الكمبيوتر فرق في ورقة منفصلة"